

Derivation of Fast Converging 2-D Periodic Green's Function Series using PML-based Eigenmodes.

Hendrik Rogier *

Abstract — A fast converging 2-D periodic Green's function for a layered medium is derived based on eigenmodes of the layered substrate terminated by a perfectly matched layer (PML). The PML-termination mimics the open configuration, yet allowing a description of the problem in terms of discrete modes. Exponential convergence is achieved by combining the PML-based modal series expansion with a truncated periodic Green's function series in the spatial domain. The efficiency of the new approach is illustrated by first comparing the convergence of this new series to the conventional series expansions for the periodic Green's function in the spectral and spatial domains. Finally the new periodic Green's function is applied to study scattering by a grid of metallic wires embedded in a dielectric slab. It is shown that the new technique results in a significant speed-up compared to existing approaches.

1 INTRODUCTION

Periodic structures with infinite extent are described very efficiently by applying the Floquet-Bloch theorem to limit the analysis to a representative unit cell. When applying integral equation techniques to describe the fields in this cell, the periodic Green's function is required to take into account the periodic character of the configuration. In general, the periodic Green's function is written as a spatial domain series or as a series of Floquet modes. Both series tend to be slowly converging for certain positions of the excitation and the observation point. Therefore, much attention has been devoted in literature to derive series expansions that converge more rapidly, mainly by combining both the spatial and the spectral domain series.

In this contribution, we propose a new formalism based on Perfectly Matched Layers (PMLs) to derive a fast converging series expansion for the 2D periodic Green's function of layered media. PMLs [1] are used to mimic the open character of the problem, in the meanwhile transforming the open layered medium into a closed waveguide configuration, leading to an efficient expansion for the Green's function in terms of a set of discrete modes of the closed waveguide. We obtain a series for the 2D periodic Green's function with exponential convergence by combining the PML-based modal

expansion with a truncated periodic Green's function series in the spatial domain [2]. The number of terms required in both series is controlled by a parameter, allowing to minimize CPU-time. An optimal choice for this parameter is based on the computational complexity of the spatial domain Green's function.

In Section 3, the new approach is applied to calculate the 2D periodic Green's function in free space. The accuracy of the new expansion is compared to the spatial domain series and the expansion in Floquet modes. It is shown that the new technique results in a significant speed-up compared to existing approaches based on a combination of the spatial and the spectral domain series. In Section 4, we study the 2D periodic Green's function in a dielectric slab. A simple integral equation approach has been implemented to study scattering from wire grids embedded in dielectric slabs. It is demonstrated that the new approach shows a significant reduction in CPU-time compared to the approach described in [3], while maintaining a comparable accuracy.

2 PML FORMALISM FOR THE PERIODIC GREEN'S FUNCTION

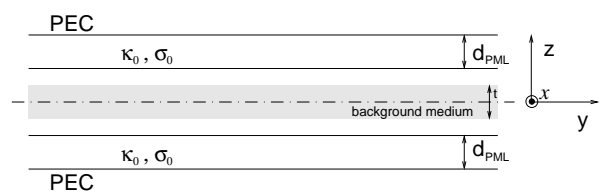


Figure 1: Stratified medium terminated by PMLs.

Assume a planar stratified medium, translation invariant in the y -direction and with all material variations in the z direction located within a region bounded by the vertical distance t . Instead of using the conventional spatial domain series expansion for a 2-D periodic array of line sources located in that medium, i.e.

$$G_{xx}^{\text{per}}(y, z|y', z') = \sum_{n=-\infty}^{\infty} G_{xx}(y, z|y' + nd, z')e^{-jnky'd}, \quad (1)$$

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we construct a parallel plate waveguide by terminating the free space with two PEC plates backed by a perfectly matched layer (PML) with thickness d_{PML} and with material parameters κ_0 and σ_0 [4], as shown in Fig. 1, so that the periodic Green's function can be expanded into a series of discrete eigenmodes:

$$G_{xx}^{\text{per}} = \sum_{n,m=-\infty}^{\infty} A_m(\beta_m, z|z') e^{-j(\beta_m|y-y'-nd|+nk_yd)}$$

with β_m the eigenvalues and with $A_m(\beta_m, z|z')$ the excitation coefficients of the eigenmodes. Assume now that $0 < y - y' < d$. Some simple manipulations result in

$$\begin{aligned} G_{xx}^{\text{per}} &= \sum_{n=-l}^l G_{xx}(y, z|y' + nd, z') e^{-jnkyd} \\ &+ \sum_{m=-\infty}^{\infty} A_m(\beta_m, z|z') \left[e^{j\beta_m(y-y')} \frac{e^{-j(\beta_m+k_y)(l+1)d}}{1 - e^{-j(\beta_m+k_y)d}} \right. \\ &\quad \left. + e^{-j(\beta_m(y-y'))} \frac{e^{-j(\beta_m-k_y)(l+1)d}}{1 - e^{-j(\beta_m-k_y)d}} \right] \end{aligned} \quad (2)$$

for an arbitrary positive value of l . The second series is the Green's function pertaining to an array of $2l+1$ sources placed inside the background medium. The parameter l can be controlled to reduce CPU-time. A larger value of l results in an increased convergence rate for the first series, at the expense of an increased number of terms in the second series. An optimal choice for l depends mainly on the computational complexity of the Green's function $G_{xx}(y, z|y' + nd, z')$.

3 FREE SPACE PERIODIC GREEN'S FUNCTION

To illustrate our formalism, we determine the Green's function $G^{\text{per}}(y, z|y', z')$ for a periodic set of line sources spaced at a distance $d = 0.02\text{m}$ and placed in free space ($k = k_0 = \frac{2\pi}{\lambda}$) at a wavelength $\lambda = 25\text{mm}$. The current on the line sources is assumed to be x -oriented. When using the PML formalism, the array of sources is surrounded by two PMLs backed by perfectly electrically conducting plates. The PMLs are placed at a distance $d_{\text{air}} = 20\text{mm}$ and their characteristics are chosen to be $d_{\text{PML}} = 5\text{mm}$, $\kappa_0 = 10$, $\frac{\sigma_0}{\omega\epsilon_0} = 5$. Fig. 2 shows the amplitude of the 2D periodic Green's function $|G_{xx}^{\text{per}}(0, z)|$, given by (2) with $A_m(\beta_m, z|z') = \frac{j(1+\delta_{m,0})}{2\tilde{d}\sqrt{k_0^2 - (\frac{2m\pi}{\tilde{d}})^2}} \cos(\frac{2m\pi}{\tilde{d}}z)$ and $\beta_m = \frac{2m\pi}{\tilde{d}}$ ($\tilde{d} = d_{\text{air}} + 2d_{\text{PML}}(\kappa_0 - j\frac{\sigma_0}{\omega\epsilon_0})$) and $\delta_{n,0}$

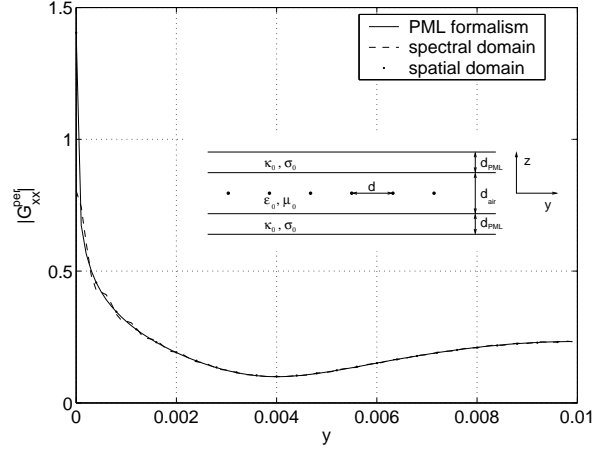


Figure 2: 2D periodic Green's function at $z = 0$.

the Kronecker delta), along $z = 0$. Because of symmetry, the plot is made over half a unit cell. The spatial domain series (1), with

$$G_{xx} = \frac{j}{4} H_0^{(2)}(k_0 \sqrt{(y - y' - nd)^2 + (z - z')^2}),$$

converges very slowly, except near $y = 0$. We used 5000 terms in the series to get a relatively accurate solution. For the spectral domain series, with $\beta_{s,n} = \frac{2n\pi}{d} - k_y$,

$$G_{xx}^{\text{per}} = - \sum_{n=-\infty}^{\infty} \frac{e^{j\beta_{s,n}(y-y') - \sqrt{\beta_{s,n}^2 - k_0^2}|z-z'|}}{2d\sqrt{\beta_{s,n}^2 - k_0^2}} \quad (3)$$

slow convergence is observed when $|y - y'|$ is small, because of the singular behavior when the observation point approaches the excitation point. We used 40 terms in the series evaluation. As one can see, the solution is not acceptable around the self-patch point $y - y' = 0$. The new formalism, which only uses 10 PML-based modes combined with 7 terms of the spatial series, provides very accurate results over the complete y -range of the unit cell.

4 PERIODIC GREEN'S FUNCTION IN A DIELECTRIC SLAB

Finally, we apply the formalism of Section 2 to determine the Green's function $G^{\text{per}}(y, z|y', z')$ for a periodic set of line sources embedded in a dielectric slab, as shown in Fig. 3. In order to apply the PML formalism, the slab together with the array of sources is surrounded by two PMLs backed by PEC plates. In order to evaluate (2), we first calculate the propagation constants of eigenmodes for the waveguide formed by the slab together with the PML, using the fast formalism described in [5].

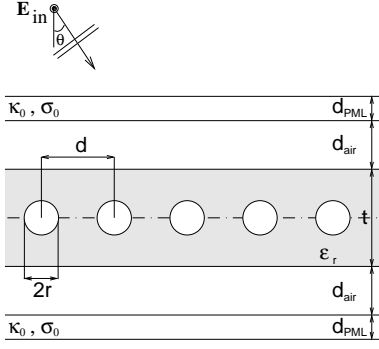


Figure 3: Wire grid in a dielectric slab.

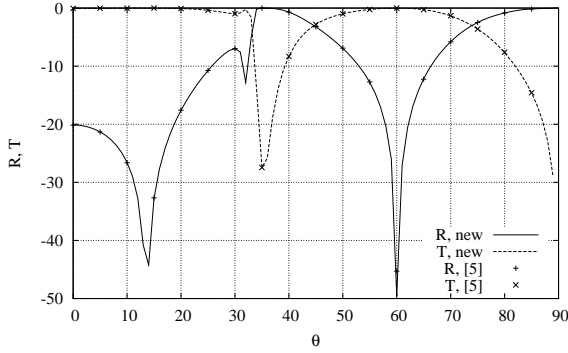


Figure 4: Power reflection and transmission coefficients for a wire grid with wire radius $r = 2\text{mm}$.

The coefficients $A_m(\beta_m, z|z')$ are then found by applying the techniques described in [6]. The spatial Green's function $G_{xx}(y, z|y', z')$ is obtained by first transforming the problem to the spectral domain and then evaluating the inverse Fourier transform, as in [7]. As this procedure is very time-consuming, we retain one term in the spatial series and choose $l = 0$. This Green's function was used as kernel function in

$$-E_x^{\text{in}} = j\omega\mu_0 \sum_i \oint_{C_i} G_{xx}^{\text{per}}(y, z; y', z') J_x(y', z') dc'$$

to model scattering by a periodic array of PEC metallic objects buried in a dielectric slab. Let us, e.g., consider a periodic grid of metallic wires (Fig. 3), buried in a dielectric slab with thickness $t = 18\text{ mm}$ and permittivity $\epsilon_r = 3.0$. The center of each wire is placed on the symmetry axes of the slab and the center-to-center spacing between the wires is chosen to be $d = 10\text{ mm}$. The PMLs are placed at a distance $d_{\text{air}} = 5\text{ mm}$ from the slab and their characteristics are chosen to be $d_{\text{PML}} = 3.5\text{ mm}$, $\kappa_0 = 15$, $\frac{\sigma_0}{\omega\epsilon_0} = 10$. The structure is excited by an x -polarized incoming plane wave, at a free-space wavelength $\lambda = 20\text{ mm}$. As the characteristics of the background medium are incorpo-

rated in the Green's function, only the currents J_x on the wires in one unit cell remain as unknowns. We compare this integral equation technique with the fast periodic kernel function for the dielectric slab with the boundary integral equation approach presented in [3]. The kernel function used there is either the free-space Green's function, accelerated by the technique described in [8], or the free-space Green's function in combination with the Floquet-Bloch condition. The latter method requires the additional discretisation of the fields at the slab-air interface, over a complete unit cell, in addition to the unknown currents on the wires. In Fig. 4 and

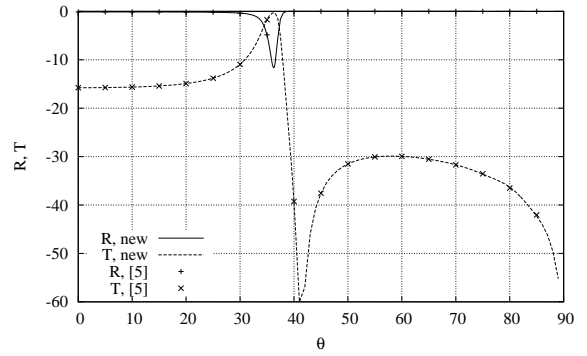


Figure 5: Power reflection and transmission coefficients for a wire grid with wire radius $r = 3\text{mm}$.

Fig. 5, the power reflection coefficient R and the power transmission coefficient T are shown for a wire grid with wire radius $r = 2\text{ mm}$ and $r = 3\text{ mm}$, respectively, as a function of the angle of incidence θ . A good agreement is seen between both the new approach and the formalism described in [3]. In Fig. 5, one observes a small loss in accuracy with the new formalism (error of the order of 0.5%) for grazing incidence (θ close to 90°), due to some small parasitic reflection at the PML. In both methods, the wire in a unit cell is modeled as a hexagone, with each side subdivided into two segments (12 unknowns to model the current J_x). In order to apply the formalism described in [3], both the horizontal walls and the vertical walls of the slab are discretised into 20 segments each. Since both the tangential electric and magnetic field components are required, this results in 160 additional unknowns. For one angle of incidence, the new implementation takes 10s of CPU time on a 2.4GHz Pentium IV, whereas the implementation described in [3] requires 2min 16s of CPU time. Finally, consider a dielectric slab with a wire grid consisting of two wires per unit cell, as shown in the inset of Fig. 6. All wires have a radius of $r = 1.5\text{ mm}$. In Fig. 6, the power reflection coefficient R and the power trans-

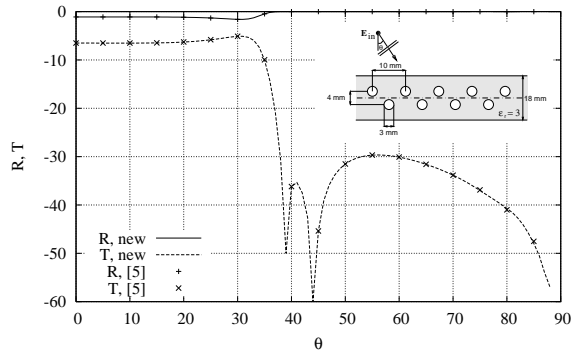


Figure 6: Power reflection and transmission coefficients for a dielectric slab with a wire grid consisting of two wires per unit cell.

mission coefficient T are shown as a function of the angle of incidence θ . A good agreement is observed between our new technique, requiring 24 unknowns to model the current, and the approach in [3], using 184 unknowns to model the fields and the currents. The new implementation is faster by a factor 6, compared to the formalism used in [3].

5 CONCLUSIONS

We introduced a new formalism based on Perfectly Matched Layers (PMLs) to derive a fast converging series expansion for the 2D periodic Green's function of layered media. A modal expansion for the waveguide formed by the layered medium terminated by PMLs was combined with a truncated periodic Green's function series in the spatial domain. The number of terms required in both series is controlled by a parameter l in order to reduce CPU-time. An optimal choice for this parameter is based on the computational complexity of the spatial domain Green's function. When the spatial domain Green's function is available in closed form, as in free space, we evaluate several terms in the spatial series ($l > 0$) in order to increase the convergence rate of the modal series. Otherwise, $l = 0$ is chosen so that only one term remains in the spatial series, since the evaluation of the spatial domain Green's function then becomes computationally involved.

Acknowledgments

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References

- [1] J. P. Béranger, "Perfectly matched layer for the FDTD solution of wave-structure interaction problems," *IEEE Trans. Antennas Propag.*, vol. 44, pp. 110–117, Jan. 1996.
- [2] H. Rogier and D. De Zutter, "A fast converging series expansion for the 2D periodic Green's function based on Perfectly Matched Layers," *IEEE Trans. Microwave Theory Tech.*, vol. 52, pp. 1199–1206, Apr. 2004.
- [3] H. Rogier, B. Baekelandt, F. Olyslager, and D. De Zutter, "The FE-BIE technique applied to some 2-D problems relevant to electromagnetic compatibility: Optimal choice of mechanisms to take into account periodicity," *IEEE Trans. Electromagnetic Compatibility*, vol. 42, pp. 246–256, Aug. 2000.
- [4] H. Derudder, F. Olyslager, and D. De Zutter, "An efficient series expansion for the 2D Green's function of a microstrip substrate using perfectly matched layers," *IEEE Microwave Guided Wave Lett.*, vol. 9, pp. 505–507, Dec. 1999.
- [5] H. Rogier, L. Knockaert, and D. De Zutter, "Fast calculation of the propagation constants of leaky and Berenger modes of planar and circular dielectric waveguides terminated by a perfectly matched layer," *Microwave Opt. Technol. Lett.*, vol. 37, pp. 167–171, May 2003.
- [6] H. Rogier and D. De Zutter, "Convergence behavior and acceleration of the Berenger and leaky modes series composing the 2D Green's function for the microstrip substrate," *IEEE Trans. Microwave Theory Tech.*, vol. 50, pp. 1696–1704, Jul. 2002.
- [7] N. Fáché, J. Van Hese, and D. De Zutter, "Generalised space domain Green's dyadic for multilayered media with special application to microwave interconnections," *Journ. of Electromagnetic Waves and Appl.*, vol. 3, no. 7, pp. 651–669, 1992.
- [8] B. Baekelandt, D. De Zutter, and F. Olyslager, "Arbitrary order asymptotic approximation of a Green's function series," *Int. Journal of Electr. and Comm.*, pp. 224–230, July 1997.

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